NONISOTHERMAL FLOW OF A POLYMERIC LIQUID UNDER A PULSATING PRESSURE GRADIENT

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A study has been made of the effects of pressure-gradient pulsations on the nonisothermal flow of a nonlinear liquid with memory in an annular channel.

Increasing flow rates is a major problem in transporting petroleum as well as polymer solutions and melts. Industrial methods are often directed to reducing the effective viscosity: heating and pulsation. The latter is related to the nonlinearity in the properties. There is considerable interest in research on this phenomenon.

It has been shown [1-4] that the flow averaged over a period can be increased for a polymeric liquid by supplementing the constant pressure difference with an oscillating component of appropriate amplitude and frequency. Three parameters influence the flow: the amplitude, the frequency, and the constant pressure difference. There is a characteristic peak on the curve for the change in relative flow rate $I = (\langle Q\rangle - Q_0)/Q_0$ ($\langle Q\rangle$ is the flow rate averaged overa a period, while Q_0 is that at constant pressure difference). I increases linearly with the square of the oscillation amplitude ξ^2 for $\xi^2 \lesssim 0.25$.

The effects of frequency on the flow rate have remained debatable. In early experiments [1] with aqueous solutions of polyacrylamide, the concentrations were from 0.1 to 2% (R-250 polyacrylamide), where I increased with ω. While the increase was slight for $\omega \lesssim 5$ rad/sec, the flow rate at higher ω increased roughly in proportion to ω^2 by factors of up to 4-5 at the maximum. Similar results were obtained in theoretical studies [5, 6]. However, other calculations for inelastic liquids such as [2, 4, 7] and for certain models of elastoviscous media [1, 3, 8-10] gave directly opposite results: the relative flow increase decreased as the pressure increased. This discrepancy was examined in [3], where it was found that the flow rate decreased as the frequency rose. The experiments of [3] were based on solutions of AR-30 polyacrylamide of concentration 1 and 1.5%, which differed from those used in [1]. The solutions also had different rheological characteristics, particularly as regards elasticity. So far, there have thus been no studies enabling one to explain the discrepancies between theory and experiment or to establish a physical mechanism for the frequency-dependent flow-rate change.

There are theoretical and experimental studies [11, 12] on the flow of a polymeric liquid under constant pressure difference in a harmonically oscillating tube. It was found that the average flow rate increased with the frequency. On transferring to a mobile coordinate system linked to the tube, the problem is reduced to that of a liquid flowing in a tube at rest under a pulsating pressure gradient.

We consider the flow of a polymeric liquid in a long coaxial cylindrical tube ($R_1 \leqq r \leqq R_2$), whose walls are kept at constant but different temperatures θ_1 and θ_2. There is a logarithmic temperature distribution $(\theta - \theta_1)/(\theta_2 - \theta_1) = (\ln r/R_1)/(\ln R_2/R_1)$ transverse to the gap.

A nonlinear integral rheological equation of state applies [13]:

$$\mathbf{T} = \int_{-\infty}^{t} m\,[t-t',\ S_{\mathbf{D}}(t')]\left[\left(1+\frac{\varepsilon}{2}\right)(\mathbf{C}_t^{-1}(t')-\mathbf{E})+\frac{\varepsilon}{2}(\mathbf{C}_t(t')-\mathbf{E})\right]dt';$$
$$m = \sum_{k=1}^{\infty} \eta_k \lambda_k^{-2} f_k(S_{\mathbf{D}}(t'))\ \exp\left[-\int_{t'}^{t} \lambda_k^{-1} g_k(S_{\mathbf{D}}(t''))\,dt''\right], \qquad (1)$$

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$$S_D^2 = 2\,\mathrm{tr}\,\mathbf{D}^2;\ \eta_k = \eta_0/\zeta(\alpha)\,k^\alpha;$$

$$\lambda_k = \lambda/k^\alpha;\ 1{,}5 \lesssim \alpha \lesssim 8.$$

The principle of time–temperature superposition [14] is used to describe the temperature dependence of the rheological parameter, which is as follows when one neglects the temperature–density correction: $\eta = \eta_S a(\theta)$; $\lambda = \lambda_S a(\theta)$; $\theta_S = 0.5(\theta_1 + \theta_2)$; $a(\theta) = \exp[(E_a/R)(1/\theta - 1/\theta_S)]$.

The liquid is driven by a pulsating pressure gradient applied at the initial instant:

$$\frac{\partial p}{\partial z}(t) = \left(\frac{\partial p}{\partial z}\right)_0 (1 + \xi \sin \omega t),\ \xi \leqslant 0{,}5. \tag{2}$$

Studies have been made on the transient response and the steady-state oscillations. We considered not only a liquid with constant properties as derived from (1) with $f_k = g_k = 1$ but also the nonlinear McDonald–Bird–Carro MBC model: $f_k = (1 + \lambda_k' S_D)/(1 + \lambda_k S_D)$; $g_k = (1 + \lambda_k' S_D)^{3/2}/(1 + \lambda_k S_D)^{1/2}$; $\lambda_k' = 0.2\lambda_k$.

The analysis was performed in dimensionless form, where we introduce the following: $El = \lambda\eta_0/\rho h^2$ the elastic number (the ratio of the maximum relaxation time to the time for a shear wave to pass across the gap in the viscous liquid), $We = \lambda V/h \equiv \lambda(\partial p/\partial z)_0 h^2/\eta_0$ the Weissenberg number (the elastic deformation in the relaxation time), $De = \omega\lambda$ the Debora number (the ratio of the maximum relaxation time to the pulsation period), $Re_\omega = \rho\omega h^2/\eta_0$ the vibrational Reynolds number, and $\delta = (R_2 - R_1)/R_1 \equiv h/R_i$ the relative gap. The thermal cirteria were the temperature difference $\nu = (\theta_1 - \theta_2)/\theta_S$ and the parameter $b = R\theta_S/E_a$ describing the temperature dependence of the viscosity. For example, if $\theta_1 = 60°C$ and $\nu = 0.15$, then $\theta_2 = 13.5°C$, while for $b = 0.05$, the viscosity near the internal wall is less than that near the outer one by a factor 20.4.

A numerical solution was obtained for the nonlinear case by means of an inexplicit conservative finite-difference scheme.

Low-amplitude oscillations superimposed on the pressure gradient produced oscillations in speed (flow rate) and stress around certain mean values corresponding to the flow produced by the constant pressure gradient applied as a pulse. The effects of El, We, and α on the flow build-up in pulsating flow are then analogous to those in cases previously considered [15].

The velocity profiles vary in phase for $\rho\omega h^2/\eta_0 \lesssim 1$ in the steady-state oscillations; the elasticity may lead to back flow with certain parameters, which occurs in the half-period corresponding to reduced pressure gradient, although the gradient remains positive throughout the period. That behavior is essentially different from that of an inelastic liquid, where return flow occurs only with negative pressure gradients. In the flow of a nonlinearly elastoviscous liquid, there is a phase shift between the pressure gradient and the flow rate.

The tangential stresses oscillate around a mean value; the largest deviations occur near the walls, and the amplitudes decrease away from them. At the point corresponding to the maximum on the stationary profile, there are virtually no oscillations even at the instants when one observes return flow, because the inertial forces are small, as are the oscillations superimposed on the pressure gradient.

The problem is linear for a liquid with constant properties, and the velocity pattern after the flow is established is the sum of the stationary velocity $v_z^{(1)}$ and the oscillating one $v_z^{(2)}$ ($\nu = 0$):

$$v_z^{(1)} = \frac{1}{4\eta_0}\left(\frac{\partial p}{\partial z}\right)_0 (r^2 - R_1^2 - (R_2^2 - R_1^2)\ln(r/R_1)/\ln(R_2/R_1));$$

$$v_z^{(2)} = \mathrm{Im}\left\{\xi\left(\frac{\partial p}{\partial z}\right)_0 [-1 + c_1 I_0(r\chi(\omega)) + c_2 K_0(r\chi(\omega))]\ \exp(i\omega t)/i\omega\rho\right\}; \tag{3}$$

$$c_1 = \frac{K_0(R_2\chi(\omega)) - K_0(R_1\chi(\omega))}{I_0(R_1\chi(\omega))K_0(R_2\chi(\omega)) - I_0(R_2\chi(\omega))K_0(R_1\chi(\omega))};$$

$$c_2 = \frac{I_0(R_1\chi(\omega)) - I_0(R_2\chi(\omega))}{I_0(R_1\chi(\omega))K_0(R_2\chi(\omega)) - I_0(R_2\chi(\omega))K_0(R_1\chi(\omega))};$$

$$\chi(\omega) = i\omega\rho/\eta(\omega).$$

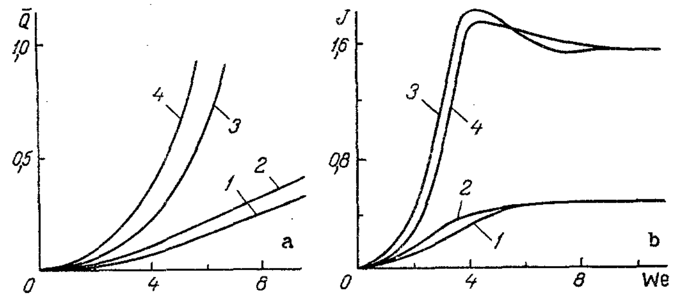

Fig. 1. Dependence of the mean flow rate (a) and the relative excess (b) on the Weissenberg number for the MBC model (De << 1, $\delta = 1$): $\alpha = 2$ (1, 2); 3 (3, 4); a – $\xi = 0$ (1, 3); 0.5 (2, 4); b – $\xi = 0.2$ (1, 3); 0.5 (2, 4).

Then $\eta_\omega = \eta_0$ for a Newtonian liquid, while $\eta_\omega = \sum_{k=1}^{\infty} \eta_k/(1 + i\lambda_k\omega)$ for an elastoviscous one.

We see from (3) that adding the oscillating quantity to the pressure difference as in (1) does not alter the mean flow rate.

For a nonlinearly elastoviscous liquid, gradient variation as in (2) also produces oscillating velocity and stress patterns,

$$v_z(r, t) = v_0(r) + \sum_{m=1}^{\infty} [v_m^{\mathrm{I}}(r) \cos m\omega t + v_m^{\mathrm{II}}(r) \sin m\omega t];$$

$$T_{rz}(r, t) = T_{rz,0}(r) + \sum_{m=1}^{\infty} [T_{rz_m}^{\mathrm{I}}(r) \cos m\omega t + T_{rz_m}^{\mathrm{II}}(r) \sin m\omega t].$$

If there is only one harmonic in the time dependence of the pressure gradient, the oscillations with frequency ω play the main parts in these Fourier series. The oscillating component of the shear velocity has amplitude $[(dv_1^{\mathrm{I}}/dr)^2 + (dv_1^{\mathrm{II}}/dr)^2]^{1/2}$, which alters the effective viscosity, which relates the stationary components of the tangential stress T_{rz_0} and the shear rate dv_0/dr, which affects the average flow rate.

There are six limiting cases for various relations between the times $1/\omega$, $\rho h^2/\eta_0$, λ:

for El >> 1

$$1)\ 1 \ll \rho\omega h^2/\eta_0 \ll \omega\lambda;\quad 2)\ \rho\omega h^2/\eta_0 \ll 1 \ll \omega\lambda;\quad 3)\ \rho\omega h^2/\eta_0 \ll \omega\lambda \ll 1$$

and for El << 1

$$4)\ 1 \ll \omega\lambda \ll \rho\omega h^2/\eta_0;\quad 5)\ \omega\lambda \ll 1 \ll \rho\omega h^2/\eta_0;\quad 6)\ \omega\lambda \ll \rho\omega h^2/\eta_0 \ll 1.$$

We consider each of these in more detail.

1. The elastic behavior is prominent (De >> 1), but as Re_ω is large, the inertial forces predominante far from the wall at high frequencies. Then $v_z^{(2)}$ varies only near the wall at distances out to $\Delta_\omega \sim \sqrt{\eta_0/\rho\omega}$, while $v^{(2)} \approx (\partial p/\partial z)_0 \xi \cos \omega t/\rho\omega$ in the rest of the cross section. The oscillating part of the shear rate with amplitude about $(\partial p/\partial z)_0 \xi/\sqrt{\rho\omega\eta_0}$ lies near the walls at distances of the order Δ_ω. Therefore, for $Re_\omega >> 1$ the effective viscosity alters only near the walls on account of the gradient pulsations, and there is no great change in the flow rate.

2. The Reynolds number is small and the elastic effects are considerable. The pulsations then alter the mean flow rate considerably, since the effective viscosity alters throughout the cross section.

3. The inertial forces are minor ($Re_\omega << 1$); the Debora number, which defines the role of the relaxation, exceeds Re_ω, but is still small, which means that the stresses relax more rapidly than the pressure gradient changes.

4. This case is analogous to the first. The behavior is determined by the inertial foces.

5. The inertial forces are large. As in cases 1 and 4, the effective viscosity alters only near the walls. The behavior is close to nonlinear viscous.

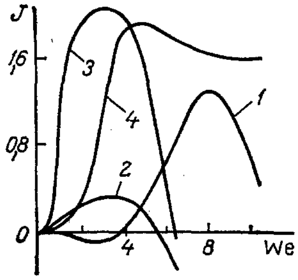

Fig. 2. Change in relative flow rate J, MBC: $\alpha = 3$, $\delta = 1$, $\nu = 0$: 1) De = 2, $Re_\omega = 0.01$; 2) 10 and 0.05; 3) 20 and 0.1; 4) quasistationary flow.

6. The pulsations are fairly effective because Re_ω is small and the elasticity is minor. As in case 3, the characteristic time for pressure gradient change is much greater than the relaxation time. When De << 1 (cases 3 and 6), one can use the formula for stationary shear flow. The instantaneous flow rate can then be calculated from

$$Q(t) = F\left[\left(\frac{\partial p}{\partial z}\right)_0 (1 + \xi \sin \omega t)\right],$$

where F defines the flow rate in a stationary state. The flow rate averaged over a period is

$$\langle Q \rangle = \frac{1}{2\pi} \int_0^{2\pi} F\left[\left(\frac{\partial p}{\partial z}\right)_0 (1 + \xi \sin \omega t)\right] dt. \quad (4)$$

In a Newtonian liquid, the stationary flow rate is linearly dependent on the gradient. With a pseudoplastic liquid, the stationary flow rate increases more rapidly than as the first power of the gradient ($F'' > 0$), while for a dilatant one, the converse applies ($F'' < 0$). Therefore, for a pseudoplastic liquid with $\xi \lesssim 1$

$$\frac{d\langle Q \rangle}{d\xi} = \frac{1}{\pi}\left(\frac{\partial p}{\partial z}\right)_0 \int_0^{\pi} \left\{F'\left[\left(\frac{\partial p}{\partial z}\right)_0 (1 + \xi \sin \omega t)\right] - F'\left[\left(\frac{\partial p}{\partial z}\right)_0 (1 - \xi \sin \omega t)\right]\right\} \sin \omega t \, dt > 0,$$

and for a dilatant one $d\langle Q\rangle/d\xi < 0$. The mean flow rate in the pulsating state is thus greater than in the stationary one with gradient $(\partial p/\partial z)_0$.

Figure 1 shows the changes in $\langle Q\rangle$ and $J = I/\xi^2$ for the MBC model calculated from (4). The We dependence of J has a characteristic peak, which is more pronounced for larger α. The calculations confirm the experimental and theoretical results on the linear dependence of I on ξ^2.

For De << 1, the behavior is close to nonlinear viscous. The results obtained for We >> 1 agree with those from the analytic formula for a power-law dependence of the viscosity with $n = 1/\alpha$ [3]: $J = (1 - 1/\alpha)\alpha^2/4$.

Particular interest attaches to the second case, where nonlinear elasticity appears, and the gradient pulsations are most effective because of the smallness of the oscillatory Reynolds number. The calculations were performed for the MBC model with $\delta = 1$, $\alpha = 3$, $\xi = 0.5$, El = 200, De = 0-20, $Re_\omega = 0.0025$-0.1, We = 2-6. The relative change in flow rate J increases as We goes from zero up to a certain value (We ≈ 4 for the parameters considered), but then it decreases, and it may enter the negative region if We is large enough (Fig. 2). This We dependence of J is characteristic of most values of De, apart from De ≈ 2. At that point, J at first takes near-zero values as We increases and then enters the negative region; J then increases with We, passes through a maximum, and then falls.

The maximal J may either exceed the quasistationary value or be less than it, in accordance with the value of the Debora number. At small De, i.e., in the region corresponding to weak elasticity, the relative change in flow rate decreases sharply as the frequency increases. Increasing the pressure gradient extends the range in Debora number for which this applied (Fig. 3). After the minimum, which may lie in the negative region, J begins to increase, which occurs in the range where the elastic behavior is more pronounced (larger De). Increase in We shifts the minimum to the right along the De axis, while J_{min} itself decreases.

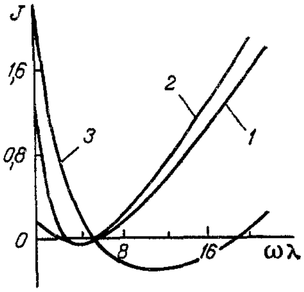

Fig. 3

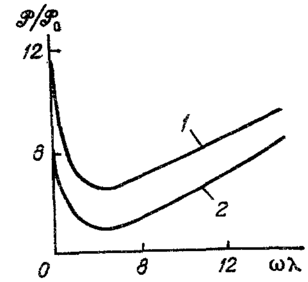

Fig. 4

Fig. 3. Effects of frequency on flow-rate change, MBC, $\alpha = 3$, $\delta = 1$, $\nu = 0$; El = 200; We = 2 (1), 4 (2), 6 (3).

Fig. 4. Power variation in pulsating flow (1) and stationary flow at the same rate (2), MBC, El = 200, We = 4, $\delta = 1$, $\xi = 0.5$, $\nu = 0$, $\alpha = 3$.

A constant temperature difference increases the flow rate averaged over a period when the outer cylinder is heated but reduces it with the inner one heated: $\langle Q\rangle|_{\nu=-0.25} = 0.45$, $\langle Q\rangle|_{\nu=0.25} = 0.20$, $\langle Q\rangle|_{\nu=0} = 0.34$ for MBC with $\alpha = 3$, $\delta = 1$, El = 200, b = 0.01, De = 10, We = 4, $\xi = 0.5$. Pulsations superimposed on nonisothermal flow cause the mean flow rate to increase in both cases: J = 3.5 if $\nu = 0.25$ and J = 1.39 if $\nu = -0.25$. The absolute value of the mean flow rate is larger in the second case than in the first. The velocity profiles vary in phase and are displaced towards the higher-temperature cylinder. The flow rates for $\nu = \mp 0.25$ at the maximum point differ by about a factor 1.5, while they are virtually equal at the minima; higher up, one finds values corresponding to the outer cylinder heated. This applies not only to the transitional stage but also to the steady-state oscillations. The tangential stresses are less at the hotter wall in nonisothermal flow.

It is important to know the amounts of power required to implemeent the pulsating and stationary flows with identical rates. The general formula for the power is

$$\mathcal{P} = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial p}{\partial z}(t)\,Q(t)\,dt.$$

Figure 4 shows the De dependence of $\mathcal{P}$; the oscillations increases the rate, but at the same time there is an increase in the power for all We and De, particularly in the range corresponding to J decreasing as De increases. The increment is reduced on the growth branch. In theoretical studies such as [2, 6, 10], it has also been found for simpified rheological equations of state that the increase in flow rate due to oscillations is accompanied by energy losses greater than in the stationary case. In a nonisothermal flow, heating the internal cylinder produces a considerable rise in I. However, here again there is an increase in power in the oscillating case.

Calculations for nonlinear equations of state with relaxation-time spectra most fully describing polymeric media thus predict a previously unknown regularity: J decreases as ω increases for $\omega\lambda$ small but increases for $\omega\lambda$ large. This agrees with experiment in the limiting cases of slight and pronounced elasticity. At present, the agreement is only qualitative, since we lack data on the relaxation times of these polymeric solutions and particularly on the dependence on shear rate.

NOTATION

r, φ, z, cylindrical coordinates; R_1, R_2, internal and external radii; t, time; $C_t(t')$, $C_t^{-1}(t')$, Cauchy and Finger finite-deformation tensors; E, unit tensor; D, deformation rate tensor; m(t), memory function; ε, model parameter; α, relaxation time spectrum parameter; $\zeta(\alpha)$, Reimann zeta function; tr, tensor sign operator; λ_k, relaxation times; λ, maximum relaxation time; η_0, initial viscosity; η_k, constants with the dimensions of viscosity; ρ, density; dp/dz, pressure gradient; T, excess stress tensor; θ, temperature; θ_S, recovery temperature; v_z, velocity component; $V = (\partial p/\partial z)_0 h^2/\eta_0$, characteristic velocity; E_α, activation energy; R, universal gas constant; ξ, fluctuation amplitude; ω, fluctuation frequency; $\mathcal{P}/\mathcal{P}_0$, dimensionless ($\mathcal{P}_0 = 2\pi R_1^2\delta\eta_0/\lambda^2$); $\bar{Q} = Q/2\pi R_1^2 V\delta$, dimensionless flow rate; $I_0(\ )$, Bessel function of imaginary argument; $K_0(\ )$, McDonald function.

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NUMERICAL MODELING OF THERMODYNAMIC PROCESSES IN A COOLED MAGNETIC-FLUID SEAL

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The results are given from numerical computations of the velocity and temperature fields in a magnetic-fluid sealing layer under one tooth of a cooled multistage magnetic-fluid seal.

The use of magnetic-fluid (MF) seals in power-generating and industrial equipment is motivated by a number of advantages that they have over conventional sealing techniques [1]. The MF in a magnetic-fluid seal moves in the narrow clearance (δ = 0.1-0.2 mm) formed by the pole piece and the rotating shaft. The performance and time to first overhaul of the high-speed MF seal depends largely on the thermodynamic processes in the sealing layer [2]. The diagnosis of the hydrodynamic and temperature fields in a MF sealing layer is complicated by the small width of the working clearance and the capacity of the MF. It is particularly important, therefore to consider numerical modeling of the thermo- and hydrodynamic processes in the working clearance of MF seals under conditions as close as possible to the actual working environment. Studies of a one-dimensional model of the MF seal have been reported in a number of papers, in which data have been obtained on the velocity and temperature fields in a MF layer whose motion in the clearance obeys Newton's friction law. It is well known, however, that concentrated MF's exhibit non-Newtonian properties, even when the carrier liquid is Newtonian [3].

In the present article we investigate the thermo- and hydrodynamic processes in a multistage MF seal with magnetic-field concentrators in the form of identical teeth on the pole piece. The position of a tooth in the pole piece is specified by the conditions for the temperature at its boundaries.

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